

Conservation Laws of Integrable Equations As Kolmogorov's Equation

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The density function $p(x, t)$ for an Ito's diffusion process satisfies Kolmogorov's forward equation, which is a conservation law of p . We shall consider some of the conservation laws of the sine-Gordon (SG) and nonlinear Schrödinger (NLS) equations and regard them as Kolmogorov's equation, to generate stochastic differential equations associated with the integrable equations. In addition, we shall analyze the derived stochastic processes numerically generated from soliton solutions, and explain the behavior of the sample paths of the stochastic variables by the equation of motion related to the nonlinear equations. We shall introduce the following results:

- By virtue of the structure of propagating wave solution one of the conservation laws of the SG equation is reduced to Kolmogorov's equation. As for the NLS equation, we can apply the method of stochastic quantization to derive the equation of motion in addition to Kolmogorov's equation.
- The behavior of the sample paths can be interpreted by using the potentials or potential-like functions of the equations of motion associated with the SG or NLS equations. The stochastic variables moves together with a attractive point generated from the solitary pulse or kinks or the equations.